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Original Research Article

FLEXURAL BUCKLING RESISTANCE OF STIFFENERS REINFORCING WALL OF A STEEL SILO

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A b s t r a c t

Stiffeners reinforcing silo's wall made from corrugated sheets are main structural elements sustaining meridional forces evoked by shear forces, source of which is friction of the material stored inside silo. Buckling resistance of stiffeners can be assessed by the method recommended in PN-EN 1993-4-1:2009/A1-2017-08E and in the draft of the Eurocode prEN 1993-4-1:2022. This method consists in determination of stiffener's resistance treated as a column resting on elastic substrate which is the susceptible wall of the silo. It means that it was assumed that the stiffness of the sheeting resists buckling displacements normal to the silo wall. The procedure recommended in mentioned Eurocodes requires determination of stiffness coefficient K on the basis of the flexibility of an equivalent arch of the span equal to the double circumferential separation between adjacent stiffeners. The stiffness coefficient K was derived in the paper using classical mechanics methods and its value was compared to the one given in mentioned Eurocodes. Careful comparative analyses resulted in the identification of errors in one of the Eurocode formulae, which could result in a significant misestimation of the buckling resistance of the stiffeners. The proposed amendment led to a full correspondence between the final formula for K derived in the paper and that proposed in the referenced Eurocodes. The errors found were reported to the coordinator of the CEN working group updating EN 1993-4-1.

Keywords: steel silo, corrugated sheeting wall, stiffener, buckling resistance, equivalent wall stiffness

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1. INTRODUCTION

A typical structural solution of steel grain silos consists of making of cylindrical part from corrugated sheets with horizontal arrangement of corrugations and introduction of additional external stiffeners (ribs) shown in Fig. 1.1. The verification of the buckling criterion of stiffeners sustaining meridional loads generated by a grain friction against wall is the key stage of a designing procedure. The amendment PN-EN 1993-4-1:2009/A1-2017-08E [1] to Eurocode EN 1993-4-1:2009 [2] which presents, between others, a modified approach to buckling resistance verification of vertical stiffeners was published in 2017. In this approach the compressed wall ribs are treated as rods resting on an elastic foundation of stiffness parameter *K*. Terminal edges of this rod are pin-ended as it was shown in Fig. 1.2. The

Fig. 1.1. Wall of a steel silo made of corrugated sheets strengthened by external stiffeners (ribs)

modification of previously functioning code recommendations consists on more precisely assessed foundation stiffness coefficient *K*. The curved wall made of corrugated or flat sheets to which stiffeners

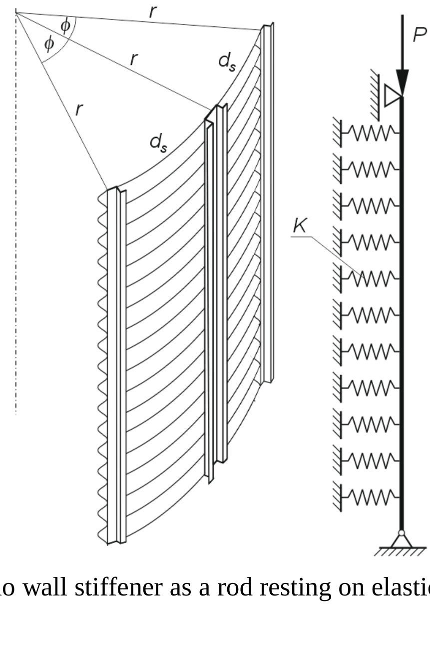

Fig. 1.2. Silo wall stiffener as a rod resting on elastic foundation

are attached constitute a kind of elastic foundation. According to the code proposal the stiffness coefficient K of this foundation should be calculated from the formula:

$$K = \frac{q}{\Delta}, \tag{1.1}$$

in which q is the load of the strip in a form of circular arch of radius r and Δ is the deflection of this arch in the place where load is applied as it was shown in Fig. 1.3. The span of the arch is equal to double distance of stiffeners separation d_s (cf. [1]) and r is the radius of the cylindrical part of a silo.

Formulae (5.58e) to (5.58h) from the amendment [1] define stiffness coefficient K for the wall made from flat steel sheets and formulae (5.74) to (5.76a) define stiffness coefficient K for walls made of horizontally corrugated steel sheets. These formulae were also published in the revised version of the standard in question prEN 1993-4-1:2022 [3]. Revision and modification procedures of this Eurocode will be completed soon and after approval by national organizations this Eurocode will be officially recommended by the European Committee for Standardization (CEN) as compulsory in whole Europe. Analytical formulae on the stiffness parameter K were derived in the presented work. The obtained formulae on K were compared with formulae inserted in the mentioned Eurocode. As a result of a careful

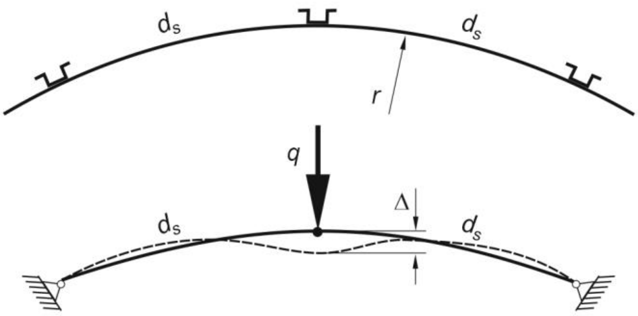

Fig. 1.3. Wall stiffness in radial direction. Eurocode concept

comparative analysis mistakes were detected in formulae (5.58h) and (5.76a) from the amendment [1]. The proposed modifications of these formula led to full compliance between the stiffness coefficient K obtained by the formula derived in this paper and the stiffness coefficient K obtained by formulae inserted in Eurocodes [1] and [3].

2. DERIVATION OF THE FORMULA ON THE STIFFNESS COEFFICIENT *K*

It was assumed in presented derivations that the cylindrical wall of the silo is corrugated according to the sine function as it was shown in Fig. 2.1. In the first step of the procedure the axial stiffness in circumferential direction and the bending stiffness with respect to the axis y shown in Fig. 2.1 of the single wave were determined. To this end its cross section A_f and the moment of inertia J_{fy} with respect to the axis y were calculated:

$$A_f = \int_0^l \sqrt{1 + \left(\frac{dz}{dy}\right)^2} \cdot t \cdot dy \; = \; t \cdot l \left(1 + \frac{\pi^2 d^2}{4l^2} - \frac{3\pi^4 d^4}{64 l^4} + \cdots\right) \cong t \cdot l \left(1 + \frac{\pi^2 d^2}{4l^2}\right), \tag{2.1}$$

$$J_{fy} = \int_0^l \sqrt{1 + \left(\frac{dz}{dy}\right)^2} \cdot t \cdot z^2 \cdot dy \;=\; \frac{t \cdot l \cdot d^2}{8}\left(1 + \frac{\pi^2 d^2}{8l^2} - \frac{\pi^4 d^4}{64 l^4} + \cdots\right) \cong \frac{1}{8} t \cdot l \cdot d^2 \left(1 + \frac{\pi^2 d^2}{8l^2}\right). \tag{2.2}$$

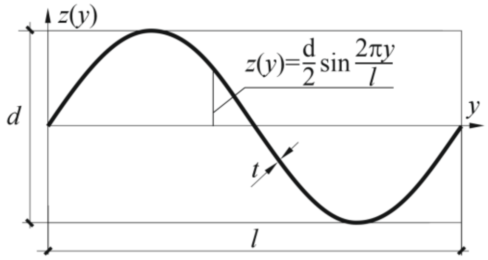

Fig. 2.1. Corrugation of a steel sheet according to the sine function

The final formulae on searched stiffnesses in reference to the wave length were obtained in the following form (cf. [2] and [3]):

$$C_y = \frac{EA_f}{l} \cong Et\left(1 + \frac{\pi^2 d^2}{4l^2}\right), \tag{2.3}$$

$$D_y = \frac{EJ_{fy}}{l} \cong \frac{1}{8} Etd^2\left(1 + \frac{\pi^2 d^2}{8l^2}\right). \tag{2.4}$$

Formulae (2.3) and (2.4) are effect of admissible simplifications (the wave length *l* is much bigger than its crest to crest distance *d* in typical corrugations of silo walls) adopted in analytical derivations of these relationships for the wave in a form of sine function. Formulae (2.3) and (2.4) are identical as their counterparts inserted in Eurocodes [2] and [3].

Stiffnesses defined by formulae (2.3) and (2.4) refer to unit length of the wall in meridional direction, hence their dimensions are [kN/m] and [kNm^2/m] for C_y and D_y respectively.

The equivalent stiffness parameter *K* can be determined analytically considering the two-hinged arch of stiffnesses C_y and D_y defined above. The searched stiffness *K* is proportional to the force acting at the crest of the arch and inversely proportional to deflection evoked by this force as it was shown in Fig. 1.3 and Fig. 2.2a.

The two-hinged arch is the case of statically indeterminate system (cf. Fig. 2.2a). The unknown value of the horizontal thrust *H* evoked by the concentrated force *P* applied at the arch crest was determined by the force method. To this end the statically determinate system shown in Fig. 2.2b was adopted. The horizontal thrust *H* is the searched unknown in this system.

In the first step of the procedure the expressions on bending moments and axial forces evoked by forces *P* and *H* for current section defined by φ angle were written. The bending moment evoked by forces *P*

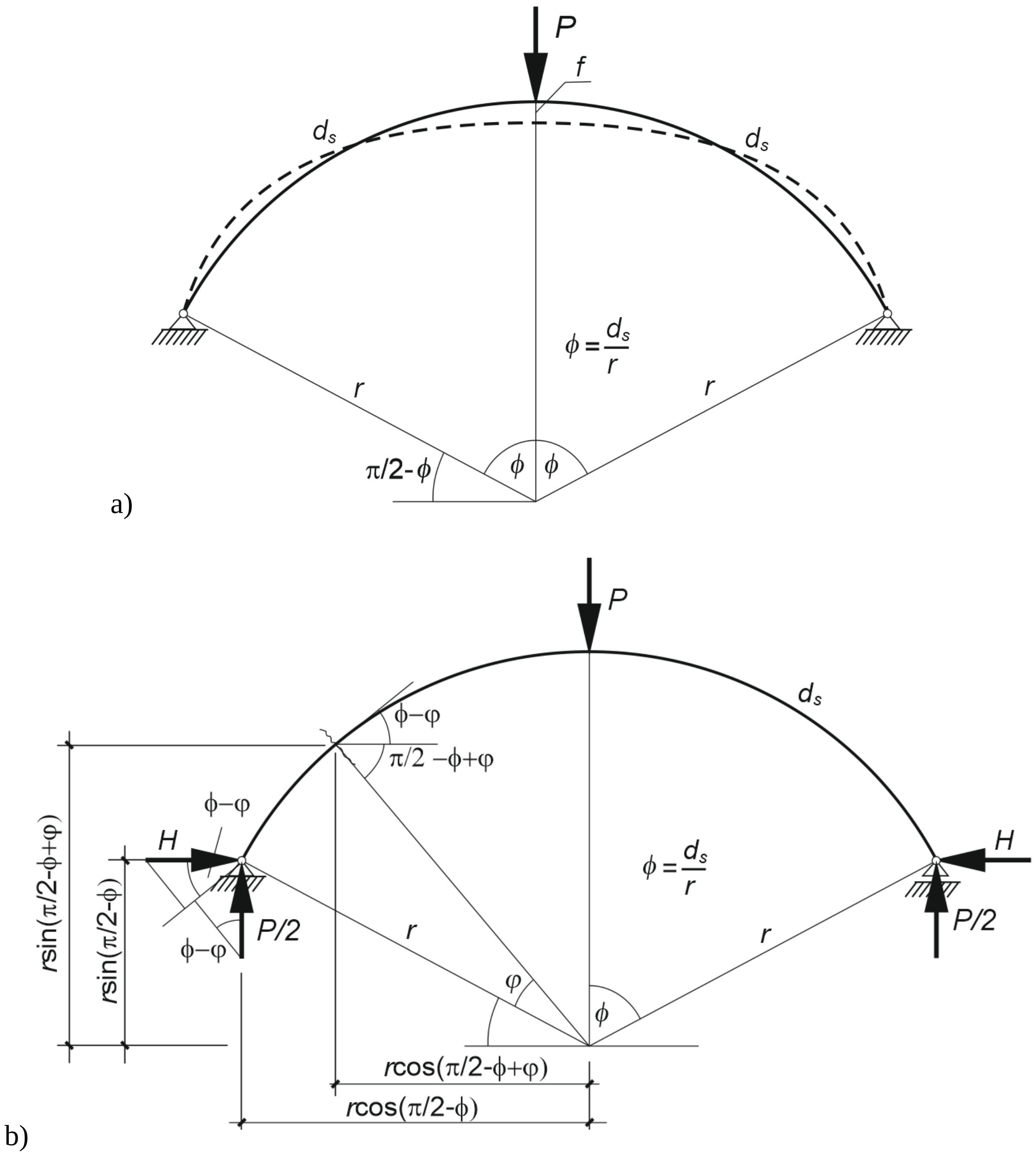

Fig. 2.2. Two hinged arch: static schema (a), schema to the horizontal thrust *H* determination (b)

and *H* in the section defined by φ angle can be defined as follows (cf. Fig. 2.2b):

$$M(P,H,\varphi) = \frac{P}{2} r \left(\cos\left(\frac{\pi}{2} - \phi\right) - cos\left(\frac{\pi}{2} - \phi + \varphi\right)\right) - Hr\left(sin\left(\frac{\pi}{2} - \phi + \varphi\right) - sin\left(\frac{\pi}{2} - \phi\right)\right), \tag{2.3}$$

and the derivative of this expression with respect to the force *H* adopts the form:

$$\frac{\partial M}{\partial H} = -r\left(sin\left(\frac{\pi}{2} - \phi + \varphi\right) - sin\left(\frac{\pi}{2} - \phi\right)\right). \tag{2.4}$$

The axial force evoked by forces *P* and *H* in the section defined by φ angle can be defined as follows (cf. Fig. 2.2b):

$$N(P, H, \varphi) = Hcos(\phi - \varphi) + \frac{P}{2} sin(\phi - \varphi)\,, \tag{2.5}$$

and the derivative of this expression with respect to the force *H* adopts the form:

$$\frac{\partial N}{\partial H} = cos(\phi - \varphi)\,. \tag{2.6}$$

Due to the boundary condition the horizontal displacement on the direction of the thrust *H* calculated according Castigliano's method should disappear and it results in the following condition:

$$\frac{2r}{D_y}\int_0^{\phi} M(P, H, \varphi)\frac{\partial M}{\partial H} d\varphi + \frac{2r}{C_y}\int_0^{\phi} N(P, H, \varphi)\frac{\partial N}{\partial H} d\varphi = 0\,, \tag{2.7}$$

which after exploiting expressions (2.3) to (2.6) leads to the following relationship on the searched horizontal thrust *H*:

$$\begin{aligned}
&\frac{2r}{D_y}\int_0^{\phi}\left[\frac{P}{2} r\left(\cos\left(\frac{\pi}{2} - \phi\right) - cos\left(\frac{\pi}{2} - \phi + \varphi\right)\right)\right. \\
&\qquad \left. - Hr\left(sin\left(\frac{\pi}{2} - \phi + \varphi\right) - sin\left(\frac{\pi}{2} - \phi\right)\right)\right]\left[-r\left(sin\left(\frac{\pi}{2} - \phi + \varphi\right) - sin\left(\frac{\pi}{2} - \phi\right)\right)\right] d\varphi \\
&+ \frac{2r}{C_y}\int_0^{\phi}\left(Hcos(\phi - \varphi) + \frac{P}{2} sin(\phi - \varphi)\right) cos(\phi - \varphi)\, d\varphi = 0\,.
\end{aligned} \tag{2.8}$$

Integrations and derivations described in the equation (2.8) were done by means of Mathematica^{TM} system [4]. As the effect of performed symbolic derivations the following formula on the searched horizontal thrust *H* was obtained:

$$H = P \cdot h(\,\phi)\ , \tag{2.9}$$

where:

$$h(\,\phi) = \frac{r^2 C_y\,[(1 - cos\phi)(1 + 3cos\phi) - \phi sin2\phi] - D_y sin^2\phi}{r^2 C_y\,[2\phi(2 + cos2\phi) - 3sin2\phi] + D_y(sin2\phi + 2\phi)}\,. \tag{2.9a}$$

The value of the deflection f in the place of the force P application can be obtained from the expression which follows from the classical Castigliano's theorem:

$$f = \frac{2r}{D_y}\int_0^{\phi} M(P,H,\varphi)\frac{\partial M}{\partial P}d\varphi + \frac{2r}{C_y}\int_0^{\phi} N(P,H,\varphi)\frac{\partial N}{\partial P}d\varphi\,. \tag{2.10}$$

Expressions on $M(P,H,\varphi)$ and $N(P,H,\varphi)$ were defined earlier by formulae (2.3) and (2.5). The formula (2.10) on deflection f under force P adopts the following form:

$$f = \\ = \frac{2Pr}{D_y}\int_0^{\phi}\left[\frac{r}{2}\left(\cos\left(\frac{\pi}{2}-\phi\right) - cos\left(\frac{\pi}{2}-\phi+\varphi\right)\right) - h(\phi)\cdot r\left(sin\left(\frac{\pi}{2}-\phi+\varphi\right) - sin\left(\frac{\pi}{2}-\phi\right)\right)\right]^2 d\varphi \\ + \frac{2Pr}{C_y}\int_0^{\phi}\left(Hcos(\phi-\varphi) + \frac{1}{2}sin(\phi-\varphi)\right)^2 d\varphi \tag{2.11}$$

Introducing the following notations:

$$S_D = h^2(2\phi cos^2\phi + \phi - 3sin\phi cos\phi) + h(2cos^2\phi + \phi sin2\phi - sin^2\phi - 2cos\phi) + \\ + \frac{3}{4}sin\phi cos\phi + \frac{1}{4}\phi + \frac{1}{2}\phi sin^2\phi - sin\phi\,, \tag{2.12}$$

$$S_C = h^2(\phi + sin\phi cos\phi) + hsin^2\phi + \frac{1}{4}(\phi - sin\phi cos\phi)\,, \tag{2.13}$$

in which $h=h(\phi)$ defined by (2.9a), the resulting formula on the deflection f evoked by the force P (after transformations of equation (2.11) made by the Mathematica™ system [4]) was obtained in the following shape:

$$f = P\frac{r^3 S_D C_y + rS_C D_y}{D_y C_y}\,, \tag{2.14}$$

and finally, the searched equivalent stiffness parameter K from the formula:

$$K = \frac{P}{f} = \frac{D_y C_y}{r^3 S_D C_y + rS_C D_y}\,. \tag{2.15}$$

3. EXAMPLES OF USING THE DERIVED FORMULA ON THE EQUIVALENT STIFFNESS PARAMETER *K*

To check the correctness of the derived formula on the equivalent stiffness parameter K, let us consider two particular cases of wall's constructions of steel silos (E=210000 N/mm^2) specified in Table 3.1.
In the 3rd row of Table 3.1 values obtained from formulae inserted in the amendment PN-EN 1993-4-1:2009/A1:2017-08 [1] to the Eurocode EN 1993-4-1:2009 [2] were presented. Comparison of these

formulae with formulae derived in the present paper makes possible to detect mistakes in formula (5.76a) on g from [1]:

Table 3.1. Considered cases of wall's constructions of steel silos

	Case 1	Case 2
1	l=76 mm, d=18 mm, t=0,75 mm, d_s=1000 mm, r=6000 mm, ϕ=0,167.	l=76 mm, d=18 mm, t=3 mm, d_s=800 mm, r=12000 mm, ϕ=0,0667.
2	C_y = 1,795·10^5 N/mm, D_y = 6,820·10^6 Nmm^2/mm, $h(\phi)$ = 4,622 eq. (2.9a), K = 1,224 N/mm^2 eq. (2.15).	C_y = 7,172·10^5 N/mm, D_y = 2,728·10^7 Nmm^2/mm, $h(\phi)$ = 10,643 eq. (2.9a), K = 2,842 N/mm^2 eq. (2.15).
3	g=4,718, g_{zm}=4,622 (error 2,07 %) K=1,209 N/mm^2, K_{zm}=1,224 N/mm^2 (error 1,21 %)	g=13,014, g_{zm}=10,643 (error 22,28 %) K=2,044 N/mm^2, K_{zm}=2,842 N/mm^2 (error 28,10 %)

$$g = \frac{D_y sin^2\phi - r^2 C_y\,[(1-cos\phi)(1+3cos\phi) - \phi sin2\phi]}{D_y(2\phi + sin2\phi) - r^2 C_y\,[2\phi(2+cos2\phi) - 3sin2\phi]}\,. \tag{3.1}$$

This formula should adopt the following form:

$$g_{zm} = \frac{r^2 C_y\,[(1-cos\phi)(1+3cos\phi) - \phi sin2\phi] - D_y sin^2\phi}{D_y(2\phi + sin2\phi) + r^2 C_y\,[2\phi(2+cos2\phi) - 3sin2\phi]}\,. \tag{3.2}$$

In the case 1 analyzed in Table 1 errors in values of g and K were relatively small. In the case 2 differences caused by indicated errors exceeded 22% in g and 28% in K respectively. The existing formula on g in a form (3.1) can even lead to a singularity in a case of a such circumferential separations of stiffeners d_s, for which denominator in formula (3.1) adopts the zero value. Plots $g(\phi)$ i $g_{zm}(\phi)$ were shown in Fig. 3.1, in which particular values for the case 2 were indicated. The mentioned singularity of the formula (3.1) on $g(\phi)$ appears for ϕ=0,0375.

It is worth mentioning that analogous mistakes are present in formula (5.58h) in the amendment PN-EN 1993-4-1:2009/A1:2017-08 [1]. This formula refers to the case of isotropic wall strengthened by external stiffeners. It follows from formula (5.76a) [1] after substitution into it:

$$C_y = Et\,,\;\; D_y = \frac{Et^3}{12}\,. \tag{3.3}$$

The correct form of the formula (5.58h) (cf. EN 1993-4-1:2009/A1:2017-08 [1]) on g in case of isotropic wall is:

$$g_{zm} = \frac{12r^2[(1-cos\phi)(1+3cos\phi) - \phi sin2\phi] - t^2 sin^2\phi}{t^2(2\phi + sin2\phi) + 12r^2[2\phi(2+cos2\phi) - 3sin2\phi]}\;, \tag{3.4}$$

while its current form is as follows:

$$g = \frac{t^2 sin^2\phi - 12r^2[(1 - cos\phi)(1 + 3cos\phi) - \phi sin2\phi]}{t^2(2\phi + sin2\phi) - 12r^2[2\phi(2 + cos2\phi) - 3sin2\phi]}\,. \tag{3.5}$$

Unfortunately, formulae on g burdened with the indicated errors were also included in the draft standard prEN 1993-4-1:2022 [3] (cf. eqn. (7.90) and eqn. (7.105) in [3]).

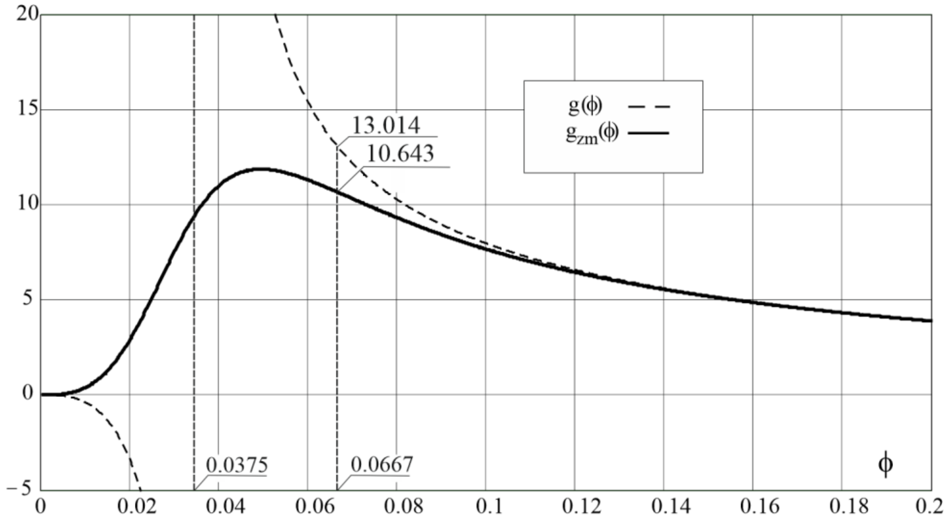

Fig. 3.1. Comparison of formulae on g and g_{zm} for the case 2 from the Table 3.1

4. RECAPITULATION AND CONCLUSIONS

Considerations inserted in this work refer to the equivalent stiffness parameter K of the steel silos wall. This quantity is required to the buckling resistance assessment of stiffeners treated in compulsory Eurocodes as a rod on an elastic foundation. From the engineering point of view its precise assessment is very important. In the procedure proposed in Eurocodes [1], [2] and [3] the stiffener is treated as a compressed pin-ended rod supported continuously along its length on elastic foundation of parameter K dependent on structural parameters of the silo's wall.

The comparative analysis of analytical formulae on K derived in the paper with formulae inserted in Eurocodes [1] and [3] have led to important findings: errors have crept into the standard formulae, resulting in a significant distortion of the value of the K parameter. Examples presented in the paper confirm, that in some cases the error made as a result of using the formula included in the standard may reach 30%. This error translates into the buckling resistance of the stiffener and the silo's wall as a whole and can significantly affect the silo's safety.

Findings made in the presented research work are very important for designers and structural engineers who apply in their practice the proposed in Eurocodes method of buckling resistance assessment of stiffeners. Information about errors present in formulae defining the equivalent stiffness parameter K should be disseminated and it was the main objective of this work.

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