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Original Research Article

RANGES OF BACKWATER CURVES IN LOWER ODRA

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A b s t r a c t

In the paper, the backwater curve ranges at the mouth of the Odra River with changing boundary conditions were analysed. The aim of the study is to determine which boundary condition, i.e. stage of lower cross-section or flow in upper cross-section, has a greater impact on the formation of the backwater curve at the mouth of the Odra River. Due to the complicated system of the Lower Odra River network (Międzyodrze and Dabie Lake), the analysis takes into consideration a section of the Odra River from a weir in Widuchowa upwards, thereby accepting as an axiom that the cross-section in Widuchowa is within the range of sea impact, regardless of other hydrological conditions.

Keywords: backwater curve, steady flow, the mouth of the Odra River, Bernoulli equation

1. INTRODUCTION

Steady, varied flow in an open channel occurs when: the cross-section of the channel is variable, the longitudinal slope is not constant, there is an obstacle in a channel or when a section of a river is in the impact zone of another river or reservoirs. The water levels in the Lower Odra River network are affected by changes in the atmospheric pressure values which arise as a result of the

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movement of low-pressure systems [6]. Additionally, the effect of wind on the surface of the sea is such that it creates drift currents causing water damming up in the coastal zone. This damming up causes wave propagation up the river. The mouth of the Odra River is located in the lowlands; the slopes of the bottom do not exceed 0.15 ‰ [1,2] (on the Bielinek - Gozdowice section) and as the river approaches Dąbie Lake, the slopes gradually decrease reaching values of 0.003 ‰ [1,2]. The section of Odra River being examined is located between two hydrological stations: Widuchowa (704.1 km of Odra) and Gozdowice (665.9 km of Odra). The calculations assume that due to the small size of the slopes, the stages in the lower Odra result exclusively from the stages of water level in the Baltic Sea. Due to this assumption, it is possible to omit the complicated calculation of flows in the lower Odra river network below the Widuchowa junction.
The main aim of the work is to analyse the calculated ranges of the backwater curve on Odra River and then determine what hydrological conditions have the greatest impact on its range.

2. MATHEMATICAL DESCRIPTION OF THE PHENOMENON

Two cross-sections of the channel in which the geometrical parameters do not change drastically can be shown in Fig. 1.

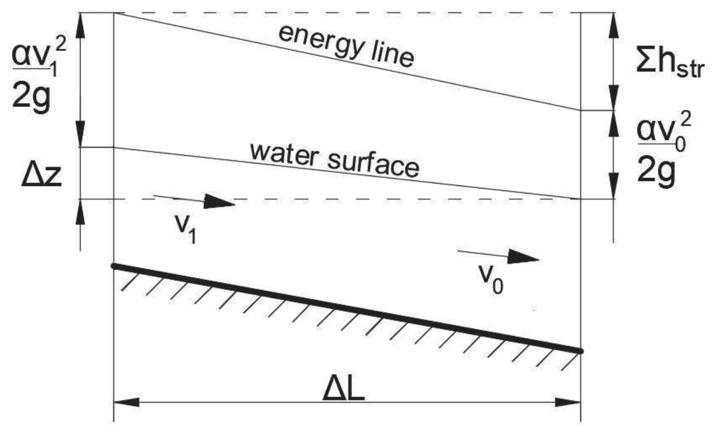

Fig. 1. The section of river contained between two cross-sections [3]

Based on the above schematic, it is possible to create a relationship between energy components in these cross-sections:

$$\sum h_{str} = \Delta z + \frac{\alpha v_1^{\ 2}}{2g} - \frac{\alpha v_0^{\ 2}}{2g} \tag{2.1}$$

where: $\sum h_{str}$ - energy loss [m]; Δz - difference of elevations of water levels between cross-sections [m]; α - Corriolis coefficient [-]; v_1, v_2 - average velocities in cross-section 1 and 0 [m/s].
Energy loss between cross-sections related to distance is called hydraulic loss. It is assumed that the average hydraulic loss is equal to the average hydraulic loss in the cross-sections, which can be determined by using the Chezy equation with Manning's formula:

$$\mathrm{I} = \frac{n^2 v^2}{R_{\mathrm{H}}^{4/3}} \tag{2.2}$$

where: n – Manning roughness [m-1/3s]; R_H – hydraulic radius [m].
Substituting average hydraulic loss to equation (1):

$$\Delta z = \Delta L \frac{I_0 + I_1}{2} + \frac{\alpha}{2g}\left(v_0^{\ 2} - v_1^{\ 2}\right) \tag{2.3}$$

In the literature one can also find the formulas for equations describing the backwater curves for the lower Odra in the form of explicit equations [7]:

$$H(x) = H_0 + \left[H(0) - H_0\right] \cdot \exp\left[-\frac{I_b - I(0)}{H(0) - H_0} x\right] \tag{2.4}$$

When analysing a river section with a given number of cross-sections, the calculations must start from the lowest cross-section (i.e. closest to the estuary). Knowing that equation (3) is an implicated equation by the variable Δz, using the method of successive approximations, the Δz value is selected so that the condition is met:

$$\left|\Delta z - \Delta L \frac{I_0 + I_1}{2} + \frac{\alpha}{2g}\left(v_0^{\ 2} - v_1^{\ 2}\right)\right| \le \varepsilon \tag{2.5}$$

where: ε – a permissible error of the closure of iterations taken as 0.001 m [-].
After successive iterations, once the value of Δz has been determined, the calculations advance to the next section and proceed in the same way as for the first pair of cross-sections, with the proviso that the previous lower cross-section will be considered as a present upper cross-section. Proceeding in this way, the

calculation is carried out upstream until the last cross-section. The range of the backwater curve is the distance between the mouth station and cross-section, where damming up is not noticeable. It is assumed that a cross-section in which the water level differs between damming and water level at normal depth does not exceed 0.01 m.

The boundary conditions for calculations were water levels in the outflow and flows in the inflow cross-section. Due to the significant impact of the sea on the water elevations in the Widuchowa section, it is not possible to create linear relationships exclusively between states and flows in this cross-section. The calculations must, therefore, be carried out for various combinations of boundary conditions. Table 1 presents the basic characteristic values for stages in the Widuchowa profile and flows for all analysed sections. These values are the basis for the creation of boundary conditions. Gauge zero (g.z.) for Widuchowa is -5.157 m a.s.l.

Table 1. Characteristic flows and stages in Widuchowa

Q [m^3/s]	W [cm g.z.]
SNQ = 288,9	SNW = 488,2
S SQ = 611,7	SSW = 601,2
SWQ = 1270,2	SWW = 644,9

Roughness coefficient (Manning's roughness) is one of the basic parameters of flows in open channels and depends on many factors. For the most part, roughness depends on the type of bank and the bottom of the channel. As shown below [4,5], the roughness coefficient, depending on hydrological conditions, may vary within a range from 0.0175 to 0.0418. Figure 2 presents changes of roughness in the flow range 223 to 1389 m^3/s.

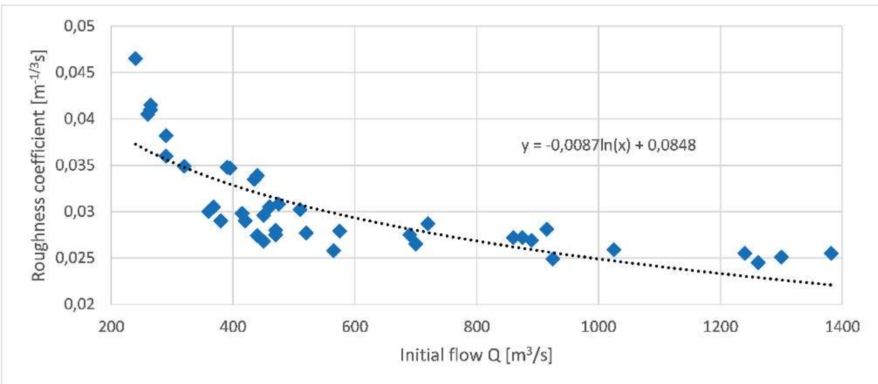

Fig. 2. Changes in Manning roughness as a function of the input flow by Kurnatowski [4,5]

In the calculations, the roughness coefficient was assumed as n = 0.030 m-1/3s. In the case of larger rivers, when the width of water surface is much larger than the depth, a hydraulic radius equals the average depth of the channel. In order to determinate the elevation of the average bottom, the inverse operation was used, i.e. given that elevation of water surface Z_{ZW} can be calculated hydraulic radius R_H, the elevation of bottom Z_D is defined as follows (Fig.3.):

$$Z_D = Z_{ZW} - R_H \qquad (2.6)$$

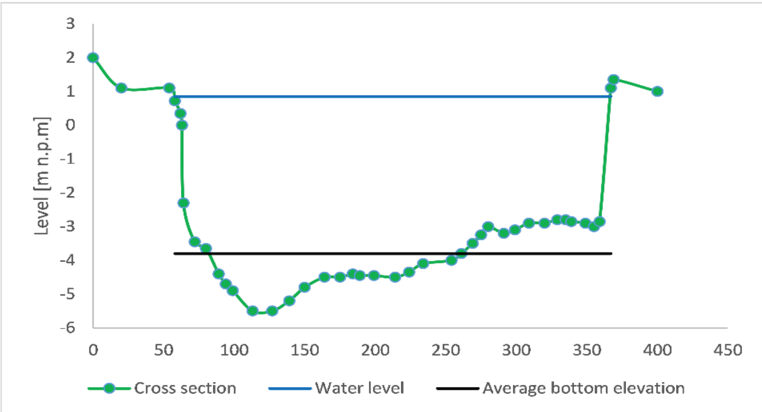

Fig. 3. Cross-section bathymetry

3. CALCULATIONS

In this paper, the calculation of ranges of backwater curves was analysed in many variants of boundary conditions. The calculations assume 16 water levels and 16 flows in the Widuchowa cross-section.
The values are presented in Table 2.

Table 2. Boundary conditions

W [cm g.z.]		Q [m³/s]		W [cm g.z.]		Q [m³/s]	
1	488.2	1	288.9	9	577.7	9	779.6
2	499.4	2	359.0	10	588.9	10	849.6
3	510.6	3	429.1	11	600.1	11	919.7
4	521.8	4	499.2	12	601.2	12	989.8
5	533.0	5	569.3	13	611.3	13	1059.9
6	544.2	6	611.7	14	622.5	14	1130.0
7	555.4	7	639.4	15	633.7	15	1200.1
8	566.6	8	709.5	16	644.9	16	1270.2

As extreme values, the average lows and average highs in stages and flows presented in Table 1 were assumed. As mentioned previously, in the Widuchowa profile the rating curve cannot be created (curve of discharge-stage relationship), therefore, calculations were made for each possible combination receiving 256 variants of boundary conditions.
As an example, Figure 4 presents the backwater curves for SSQ flows and characteristic stages.

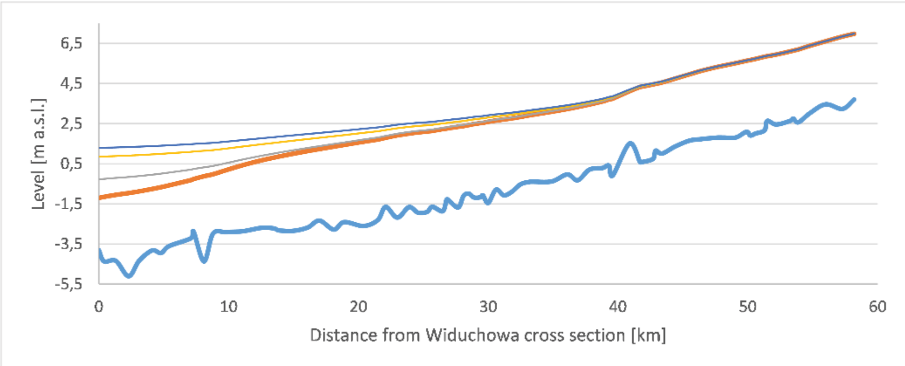

Fig. 4. Backwater curves for SSQ flows. Light blue – river bottom; orange – normal depth (without damming); grey - curve for Q = 599 m^3/s and W = -0.275 m. a.s.l.; gold - curve for Q = 599 m^3/s and W = 0.855 m. a.s.l.; blue - curve for Q = 599 m^3/s and W = 1.292 m. a.s.l.

The calculation results of backwater curves are presented in Table 3; the graphics interpretation can be seen in Figure 5. The maximum range of the backwater curve is obtained for the maximum level in the lower cross-section with the maximum flow, while the minimum range of the backwater curve can be observed for maximum flow and minimum water level in the lower cross-section. The table also presents the information for maximum, minimum, and mean ranges of backwater curves for one constant value of the boundary conditions (for constant stages or flows) with various values of the second condition.
Analysing the values in Table 3 (assuming constant values of stages in the lower cross-section and variable flow values), it can be seen that for constant stages in a lower station, the maximum range of backwater curve is not always observed with extreme flow values. For example, with the 544 cm stage, the maximum calculated range of the backwater curve is 46.5 km at a flow of 639.4 m^3/s.

With the increase in water levels in the lower cross-section, the maximum ranges are obtained at higher flow values. Differences between the maximum and minimum ranges of the backwater curve at constant stages higher than 499 cm are very small; the mean is 5.4 km. At small stages, the differences are significant and amount to 26.2 km at W = 488 cm, and 14.0 km at W = 499 cm. When analysing the ranges at constant values of flows and variable stages in the lower cross-section, one can notice a constant increase in the values of ranges together with the increase of stages. The differences between maximum and minimum ranges of backwater are higher at higher flows. According to the simulation, the maximum range of the backwater curve is obtained for maximum stages and maximum flow, achieving a range of 57.8 km above the Widuchowa junction. The smallest range was obtained with the minimum water level in the cross-section and the maximum flow; the range of backwater curve under these boundary conditions is 15.6 km – near Krajnik Gorny. Depending on the hydrological conditions, the range of the sea backwater curve on the Odra River may fluctuate over a distance of 42 km.

Table 3. Calculated ranges of backwater curves at various boundary conditions [km]

Q [m³/s] W [cmg.z.]		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	MIN	MAX	Mean	MAX - MIN
		289	359	429	499	569	612	639	709	780	850	920	990	1060	1130	1200	1270				
1	488	42.3	41.1	40.7	40.7	41.1	41.4	41.5	41.9	41.9	41.4	40.1	37.9	34.5	29.8	23.6	15.6	15.6	42.3	37.2	26.7
2	499	42.4	41.5	41.2	41.2	41.4	41.6	41.7	42.1	42.2	42.1	41.6	39.5	38.8	36.3	32.8	28.3	28.3	42.4	39.7	14.1
3	511	41.7	42.2	42.5	42.8	43.0	43.1	43.1	43.0	42.8	42.5	42.0	41.3	40.4	39.4	38.1	36.6	36.6	43.1	41.5	6.4
4	522	41.8	42.7	43.5	44.0	44.3	44.4	44.5	44.4	44.2	43.9	43.5	42.9	42.3	41.5	40.7	39.9	39.9	44.5	43.0	4.6
5	533	42.1	43.3	44.2	44.9	45.4	45.6	45.7	45.7	45.6	45.4	45.0	44.6	44.0	43.4	42.8	42.1	42.1	45.7	44.4	3.7
6	544	43.0	44.2	45.1	45.8	46.2	46.4	46.5	46.5	46.4	46.2	45.9	45.6	45.3	45.0	44.7	44.5	43	46.5	45.5	3.6
7	555	43.7	44.8	45.7	46.4	46.9	47.1	47.2	47.4	47.4	47.3	47.2	46.9	46.6	46.2	45.8	45.3	43.7	47.4	46.4	3.7
8	567	44.5	45.5	46.4	47.0	47.5	47.7	47.8	48.1	48.2	48.2	48.1	48.0	47.9	47.8	47.7	47.6	44.5	48.2	47.4	3.7
9	578	45.5	46.5	47.3	48.0	48.5	48.8	48.9	49.2	49.4	49.5	49.6	49.5	49.5	49.3	49.2	49.0	45.5	49.6	48.6	4.1
10	589	45.9	47.2	48.2	49.0	49.6	49.9	50.1	50.4	50.5	50.6	50.6	50.6	50.5	50.5	50.4	50.4	45.9	50.6	49.7	4.7
11	600	46.4	47.8	49.0	49.9	50.6	50.9	51.1	51.5	51.8	51.9	52.0	52.1	52.1	52.1	52.2	52.3	46.4	52.3	50.9	5.9
12	601	46.4	47.8	48.9	49.9	50.6	51.0	51.2	51.7	52.0	52.2	52.4	52.4	52.4	52.4	52.4	52.4	46.4	52.4	51.0	6.0
13	611	47.6	49.0	50.1	51.1	51.8	52.2	52.4	52.8	53.1	53.3	53.4	53.4	53.4	53.4	53.4	53.4	47.6	53.4	52.1	5.9
14	623	48.1	49.6	50.8	51.8	52.6	53.0	53.2	53.7	54.0	54.3	54.4	54.5	54.6	54.7	54.8	55.0	48.1	55	53.1	6.9
15	634	49.3	50.7	51.9	52.8	53.5	53.9	54.1	54.5	54.7	55.0	55.1	55.3	55.4	55.6	55.9	56.3	49.3	56.3	54.0	7.0
16	645	50.0	51.5	52.7	53.6	54.4	54.7	54.9	55.3	55.6	55.8	56.0	56.2	56.4	56.7	57.1	57.6	50	57.6	54.9	7.6
MIN		41.7	41.1	40.7	40.7	41.1	41.4	41.5	41.9	41.9	41.4	40.1	37.9	34.5	29.8	23.6	15.6				
MAX		50.0	51.5	52.7	53.6	54.4	54.7	54.9	55.3	55.6	55.8	56.0	56.2	56.4	56.7	57.1	57.6				
Mean		45.0	46.0	46.8	47.4	48.0	48.2	48.4	48.6	48.8	48.7	48.6	48.2	47.8	47.1	46.3	45.4				
MAX - MIN		8.3	10.4	12.0	12.9	13.3	13.3	13.4	13.4	13.7	14.5	15.9	18.3	21.9	26.9	33.5	42.0				

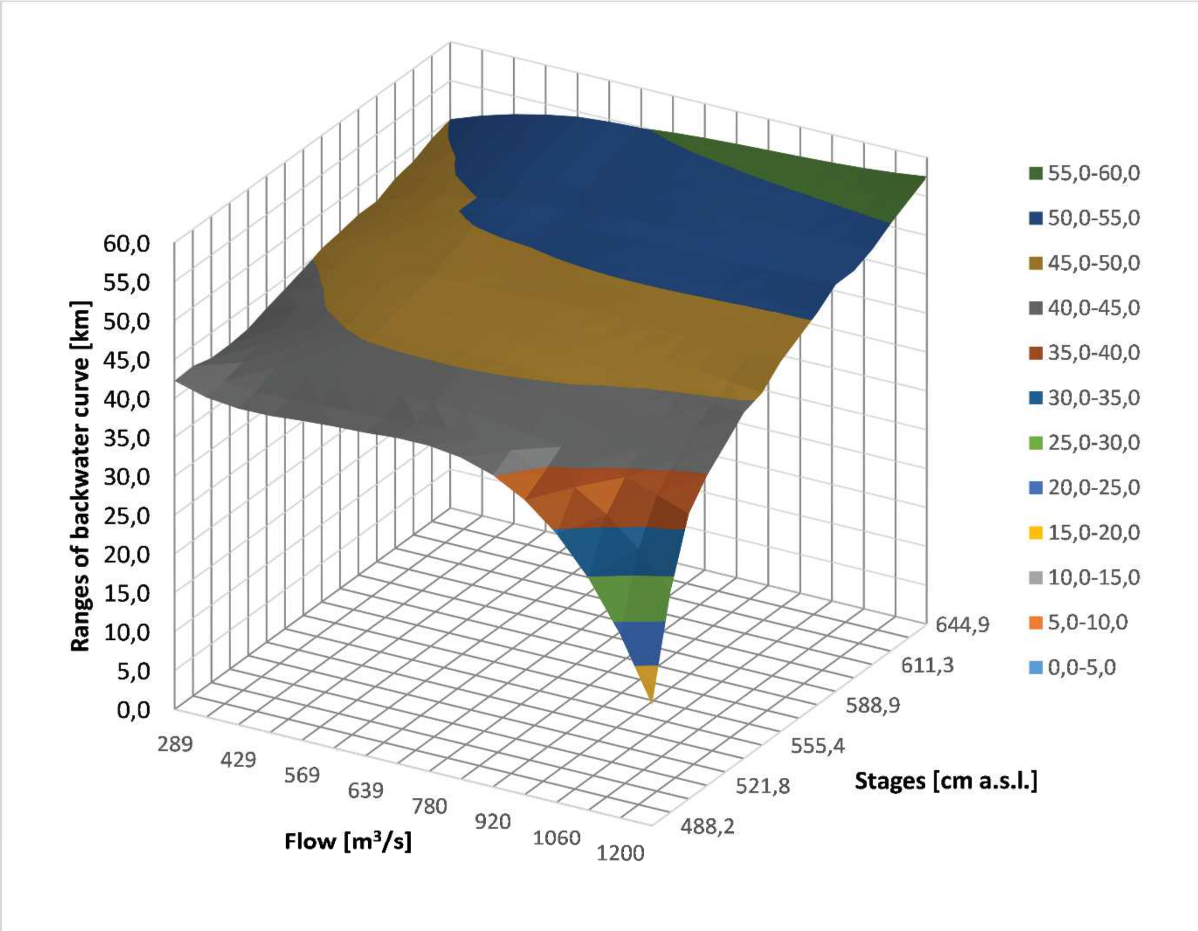

Fig. 5. Calculated ranges of backwater curves at various boundary conditions

4. CONCLUSIONS

- It is possible to create a model of backwater curve using Bernoulli's equations for the estuary section of the Odra River;
- Differences between the maximum and minimum ranges at various flows and constant water levels at the lower cross-section are small and average 5.3 km; Differences between the maximum and minimum ranges at constant water level and various flows increase as flow increases;
- As can be seen, the smallest ranges of backwater curves are observed in the zone of high water stages with the simultaneous occurrence of low flows. This is probably due to the low kinetic energy of the stream (part of equation 3: $v_0^2 - v_1^2$), which is responsible for curvature of the curve when it reaches the normal depth;
- Depending on the hydrological conditions, the ranges of backwater curves in the lower Odra river section may range from 15.6 to almost 58 km above the Widuchowa weir, which is equivalent to over 150 km of sea influence from the Swinoujscie profile, which confirms similar considerations set out in [8];
- The analyses clearly show that lower boundary conditions have a greater impact on backwater curve formation, meaning that the sea stages have the highest influence on the lengths of the backwater ranges in the estuary section of the Odra River;
- The presented results of the ranges of backwater curves can be used by engineers in preliminary studies for training works in the lower Odra.

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ZASIĘGI KRZYWEJ SPIĘTRZENIA DOLNEJ ODRY

S t r e s z c z e n i e

W pracy poddano analizie zasięgi krzywych spiętrzenia w ujściowym odcinku Odry przy zmiennych warunkach brzegowych. Celem pracy jest określenie, który warunek brzegowy tj. napełnienie stanowiska dolnego czy wartość przepływu w rzece ma większy wpływ na formowanie się krzywej spiętrzenia w ujściowym odcinku Odry. Ze względu na skomplikowany układ sieci rzecznej dolnej Odry (tj. Międzyodrze i jezioro Dąbie) do analizy przyjęto odcinek Odry od jazu w Widuchowej w górę koryta, zarazem przyjmując w obliczeniach, że stany wody w przekroju Widuchowa determinowane są wyłącznie stanami morza.

Słowa kluczowe: krzywa spiętrzenia, ruch ustalony jednostajny, ujście Odry, równanie Bernoulliego

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